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THEORETICAL RESEARCHES ABOUT DYNAMIC FORCES TRANSMITTED TO THE STRUCTURE THROUGH VISCOUS-ELASTIC BEARINGS BY THE RIGID BODY WITH SYMMETRIES

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Abstract: The goal of this theoretical study is to establish the analytical expressions of the transmitted forces to the foundation trough the elastic or viscous-elastic bearings by the rigid body, when this one is subjected to some different types of harmonical forces or/and couples.

1. INTRODUCTION

The dynamic analyze of the mechanical equipment driven by vibration or subjected to determinist or stochastic vibration consist in establishing the system's response function of its structural features and of the bearings.

Knowing the dynamic forces transmitted by the equipment is very useful for the designers of the equipment bearing and foundations.

2. THE FORCES TRANSMITTED BY THE RIGID BODY WITH VISCOUS-ELASTIC BEARINGS

Taking into consideration the model of the rigid body from figure 1 of [1] with n elastic bearings and m viscous bearings.

The elastic strains of the bearing from point $\boldsymbol{M_i}(x_i,y_i,z_i)$ are equal to the displacements of this point on three directions [1]:

$$\bar{u}_i \begin{cases} u_{ix} = X + z_i\varphi_y - y_i\varphi_z \\ u_{iy} = Y + x_i\varphi_z - z_i\varphi_x \\ u_{iz} = Z + y_i\varphi_x - x_i\varphi_y \end{cases} \tag{1}$$

The transmitted force to the structure by the elastic bearing from point $\boldsymbol{M_i}$ is

$$\bar{F}_i^e = k_{ix}u_{ix}\bar{i} + k_{iy}u_{iy}\bar{j} + k_{iz}u_{iz}\bar{k}\ , \tag{2}$$

where $k_i(k_{ix},k_{iy},k_{iz})$ are the elastic coefficients of the bearing.

The modulus of the force (2) is:

$$F_i^e = \sqrt{(k_{ix}u_{ix})^2 + (k_{iy}u_{iy})^2 + (k_{iz}u_{iz})^2} \tag{3}$$

The strain velocities of the viscous bearing from point $\boldsymbol{N_j}(x_j,y_j,z_j)$ are equal to the three direction velocities of the point $\boldsymbol{N_j}$ like these:

$$\bar{v}_j \begin{cases} v_{jx} = \dot{X} + z_j\omega_y - y_j\omega_z \\ v_{jy} = \dot{Y} + x_j\omega_z - z_j\omega_x \\ v_{jz} = \dot{Z} + y_j\omega_x - x_j\omega_y \end{cases} \tag{4}$$

The transmitted force by the viscous bearing is

$$\bar{F}_j^v = c_{jx}v_{jx}\bar{i} + c_{jy}v_{jy}\bar{j} + c_{jz}v_{jz}\bar{k}\ , \tag{5}$$

where $c_j(c_{jx}, c_{jy}, c_{jz})$ are the damping coefficients of the bearing.

The modulus of the viscous force transmitted by the point $\boldsymbol{N_j}$ is:

$$F_j^v = \sqrt{(c_{jx}v_{jx})^2 + (c_{jy}v_{jy})^2 + (c_{jz}v_{jz})^2} \tag{6}$$

3. THE TRANSMITTED FORCES TO THE STRUCTURE BY THE RIGID BODY WITH ELASTIC BEARINGS

Most of technological equipment or machines modeled like mechanical systems which are driven by vibrators or disturbed by unexpected vibration have natural frequencies much lower than the vibration frequency. Taking into consideration the influence of the damping to the amplitude of forced vibration which is decreasing alongside the increasing of the frequency of the excitation, it can considerate good enough the model of the rigid body with elastic bearing for the dynamic analyze of the transmitted forces to the structure.

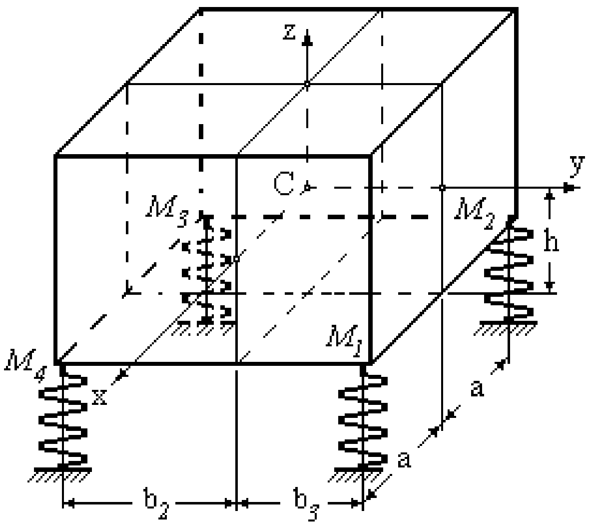

Fig. 1. Rigid body with four elastic bearings and a vertical axis of symmetry

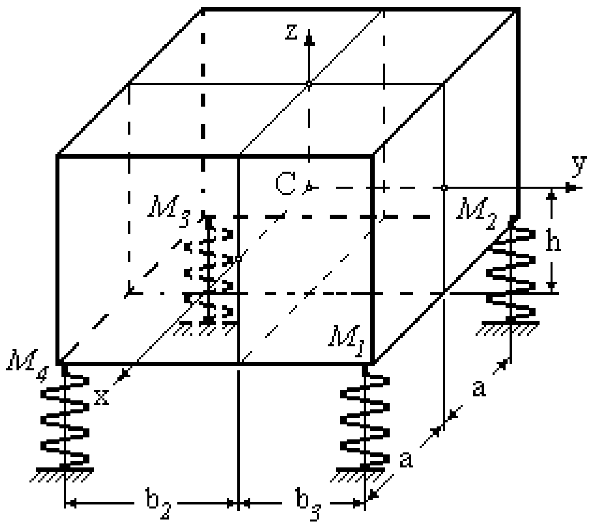

Fig. 2. Rigid body with four elastic bearings and a vertical-longitudinal plane of symmetry

3.1. Rigid body with a vertical axis of symmetry

The model of the rigid body with a vertical axis of symmetry is shown in the ***figure 1***. The coordinate of the four points with elastic bearings are:

$$\begin{cases} M_1(a, b, -h) \\ M_2(-a, b, -h) \\ M_3(-a, -b, -h) \\ M_4(a, -b, -h) \end{cases} \tag{7}$$

The steady-state forced vibration of the rigid body excited by harmonical forces and/or couples are harmonical too, the analytical expressions of the amplitudes being given acc. [5]. Because the bearings are considered only elastic, the forced vibration of the rigid body are in phase or in phase opposition to the harmonical excitation; the same variations have the displacements of the points where are the elastic bearings. That’s why the elastic strains of the bearings on the three directions have the maximum values in the same time; the values of the transmitted forces through elastic bearings are obtaining by multiplying the amplitudes of elastic strains with elastic coefficients.

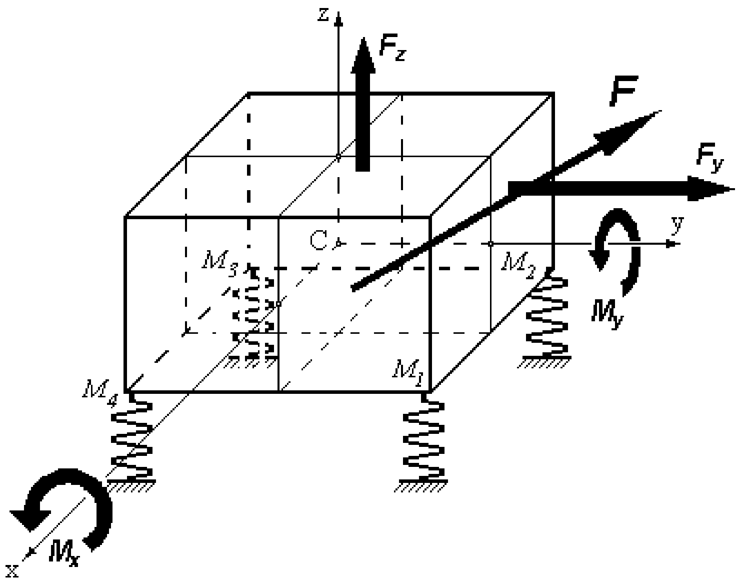

Fig. 3. Rigid body with elastic bearings, excited by some harmonical forces and couples

Considering the harmonical excitation from the ***figure 3***, the elastic strains of the springs and the modulus of maximum transmitted force to the structure are:

►*Excitation by rolling couple M_x*

●the elastic strains of the springs are:

$$u_{1x} = u_{2x} = u_{3x} = u_{4x} = 0 \tag{8a}$$

$$u_{1y} = u_{2y} = u_{3y} = u_{4y} = \left(A_Y + hA_{\varphi_x}\right)\sin\omega t \tag{8b}$$

$$u_{1z} = u_{2z} = -u_{3z} = -u_{4z} = bA_{\varphi_x}\sin\omega t \tag{8c}$$

●the modulus of maximum transmitted forces are

$$F_{1x} = F_{2x} = F_{3x} = F_{4x} = 0 \tag{9a}$$

$$F_{1y} = F_{2y} = F_{3y} = F_{4y} = k_y\left|A_Y + hA_{\varphi_x}\right| \tag{9b}$$

$$F_{1z} = F_{2z} = F_{3z} = F_{4z} = k_z b\left|A_{\varphi_x}\right| \tag{9c}$$

where A_Y and A_{φ_x} have the expressions (2.4.52)-(2.4.54) from [5].

►*Excitation by pitching couple M_y*

●the elastic strains of the springs are:

$$u_{1x} = u_{2x} = u_{3x} = u_{4x} = \left(A_X + hA_{\varphi_y}\right)\sin\omega t \tag{10a}$$

$$u_{1y} = u_{2y} = u_{3y} = u_{4y} = 0 \tag{10b}$$

$$u_{1z} = -u_{2z} = -u_{3z} = u_{4z} = -aA_{\varphi_y}\sin\omega t \tag{10c}$$

●the modulus of maximum transmitted forces are

$$F_{1x} = F_{2x} = F_{3x} = F_{4x} = k_x\left|A_X + hA_{\varphi_y}\right| \tag{11a}$$

$$F_{1y} = F_{2y} = F_{3y} = F_{4y} = 0 \tag{11b}$$

$$F_{1z} = F_{2z} = F_{3z} = F_{4z} = k_z a\left|A_{\varphi_y}\right| \tag{11c}$$

where A_X and A_{φ_y} have the expressions (2.4.59)-(2.4.61) from [5].

►*Excitation by eccentric vertical force F_z*

●the elastic strains of the springs are:

$$u_{1x} = u_{2x} = u_{3x} = u_{4x} = \left(A_X - hA_{\varphi_y}\right)\sin\omega t \tag{12a}$$

$$u_{1y} = u_{2y} = u_{3y} = u_{4y} = \left(A_Y + hA_{\varphi_x}\right)\sin\omega t \tag{12b}$$

$$u_{1z} = \left(A_Z + bA_{\varphi_x} - aA_{\varphi_y}\right)\sin\omega t \tag{12c}$$

$$u_{2z} = \left(A_Z + bA_{\varphi_x} + aA_{\varphi_y}\right)\sin\omega t \tag{12d}$$

$$u_{3z} = \left(A_Z - bA_{\varphi_x} + aA_{\varphi_y}\right)\sin\omega t \tag{12e}$$

$$u_{4z} = \left(A_Z - bA_{\varphi_x} - aA_{\varphi_y}\right)\sin\omega t \tag{12f}$$

●the modulus of maximum transmitted forces are

$$F_{1x} = F_{2x} = F_{3x} = F_{4x} = k_x\left|A_X - hA_{\varphi_y}\right| \tag{13a}$$

$$F_{1y} = F_{2y} = F_{3y} = F_{4y} = k_y\left|A_Y + hA_{\varphi_x}\right| \tag{13b}$$

$$F_{1z} = k_z\left|A_Z + bA_{\varphi_x} - aA_{\varphi_y}\right| \tag{13c}$$

$$F_{2z} = k_z\left|A_Z + bA_{\varphi_x} + aA_{\varphi_y}\right| \tag{13d}$$

$$F_{3z} = k_z\left|A_Z - bA_{\varphi_x} + aA_{\varphi_y}\right| \tag{13e}$$

$$F_{4z} = k_z\left|A_Z - bA_{\varphi_x} - aA_{\varphi_y}\right| \tag{13f}$$

where A_X, A_Y, A_Z, A_{φ_x} and A_{φ_y} have the expressions (2.4.66)-(2.4.68), (2.4.70)-(2.4.72), (2.4.75) from [5].

►*Excitation by eccentric longitudinal force F_y*

●the elastic strains of the springs are:

$$u_{1x} = u_{2x} = -u_{3x} = -u_{4x} = -bA_{\varphi_z}\sin\omega t \tag{14a}$$

$$u_{1y} = u_{4y} = \left(A_Y + aA_{\varphi_z} + hA_{\varphi_y}\right)\sin\omega t \tag{14b}$$

$$u_{2y} = u_{3y} = \left(A_Y - aA_{\varphi_z} + hA_{\varphi_y}\right)\sin\omega t \tag{14c}$$

$$u_{1z} = u_{2z} = -u_{3z} = -u_{4z} = bA_{\varphi_x}\sin\omega t \tag{14d}$$

●the modulus of maximum transmitted forces are

$$F_{1x} = F_{2x} = F_{3x} = F_{4x} = k_x b\left|A_{\varphi_z}\right| \tag{15a}$$

$$F_{1y} = F_{4y} = k_y\left|A_Y + aA_{\varphi_z} + hA_{\varphi_y}\right| \tag{15b}$$

$$F_{2y} = F_{3y} = k_y\left|A_Y - aA_{\varphi_z} + hA_{\varphi_y}\right| \tag{15c}$$

$$F_{1z} = F_{2z} = F_{3z} = F_{4z} = k_z b\left|A_{\varphi_x}\right| \tag{15d}$$

where A_Y, A_{φ_x} and A_{φ_z} have the expressions (2.4.80)-(2.4.82), (2.4.85) from [5].

►*Excitation by inclined longitudinal force F*

●the elastic strains of the springs are:

$$u_{1x} = u_{2x} = \left(A_X - hA_{\varphi_y} - bA_{\varphi_z}\right)\sin\omega t \tag{16a}$$

$$u_{3x} = u_{4x} = \left(A_X - hA_{\varphi_y} + bA_{\varphi_z}\right)\sin\omega t \quad (16b)$$

$$u_{1y} = u_{4y} = \left(A_Y + aA_{\varphi_z} + hA_{\varphi_x}\right)\sin\omega t \quad (16c)$$

$$u_{2y} = u_{3y} = \left(A_Y - aA_{\varphi_z} + hA_{\varphi_x}\right)\sin\omega t \quad (16d)$$

$$u_{1z} = \left(A_Z + bA_{\varphi_x} - aA_{\varphi_y}\right)\sin\omega t \quad (16e)$$

$$u_{2z} = \left(A_Z + bA_{\varphi_x} + aA_{\varphi_y}\right)\sin\omega t \quad (16f)$$

$$u_{3z} = \left(A_Z - bA_{\varphi_x} - aA_{\varphi_y}\right)\sin\omega t \quad (16g)$$

$$u_{4z} = \left(A_Z - bA_{\varphi_x} + aA_{\varphi_y}\right)\sin\omega t \quad (16h)$$

●the modulus of maximum transmitted forces are

$$F_{1x} = F_{2x} = k_x\left|A_X - hA_{\varphi_y} - bA_{\varphi_z}\right| \quad (17a)$$

$$F_{3x} = F_{4x} = k_x\left|A_X - hA_{\varphi_y} + bA_{\varphi_z}\right| \quad (17b)$$

$$F_{1y} = F_{4y} = k_y\left|A_Y + aA_{\varphi_z} + hA_{\varphi_x}\right| \quad (17c)$$

$$F_{2y} = F_{3y} = k_y\left|A_Y - aA_{\varphi_z} + hA_{\varphi_x}\right| \quad (17d)$$

$$F_{1z} = k_z\left|A_Z + bA_{\varphi_x} - aA_{\varphi_y}\right| \quad (17e)$$

$$F_{2z} = k_z\left|A_Z + bA_{\varphi_x} + aA_{\varphi_y}\right| \quad (17f)$$

$$F_{3z} = k_z\left|A_Z - bA_{\varphi_x} - aA_{\varphi_y}\right| \quad (17g)$$

$$F_{4z} = k_z\left|A_Z - bA_{\varphi_x} + aA_{\varphi_y}\right| \quad (17h)$$

where A_X, A_Y, A_Z, A_{φ_x} A_{φ_y} and A_{φ_z} have the expressions (2.4.97)-(2.4.99), (2.4.101)-(2.4.103), (2.4.105), (2.4.107) from [5].

3.2. Rigid body with a vertical-longitudinal plane of symmetry

The ***figure 2*** shows the model of the rigid body with a vertical-longitudinal plane of symmetry and four elastic bearings; the points where these bearings are have the coordinates:

$$\begin{cases} M_1(a, b_3, -h) \\ M_2(-a, b_3, -h) \\ M_3(-a, -b_2, -h) \\ M_4(a, -b_2, -h) \end{cases} \quad (18)$$

Taking into consideration the same harmonical excitation like in §3.1 (figure 3), the elastic strains of the springs and the maximum values of the transmitted forces are:

►*Excitation by rolling couple M_x*

●the elastic strains of the springs are:

$$u_{1x} = u_{2x} = u_{3x} = u_{4x} = 0 \quad (19a)$$

$$u_{1y} = u_{2y} = u_{3y} = u_{4y} = \left(A_Y + hA_{\varphi_x}\right)\sin\omega t \quad (19b)$$

$$u_{1z} = u_{2z} = \left(A_Z + b_3 A_{\varphi_x}\right)\sin\omega t \quad (19c)$$

$$u_{3z} = u_{4z} = \left(A_Z - b_2 A_{\varphi_x}\right)\sin\omega t \quad (19d)$$

●the modulus of maximum transmitted forces are

$$F_{1x} = F_{2x} = F_{3x} = F_{4x} = 0 \quad (20a)$$

$$F_{1y} = F_{2y} = F_{3y} = F_{4y} = k_y\left|A_Y + hA_{\varphi_x}\right| \quad (20b)$$

$$F_{1z} = F_{2z} = k_z\left|A_Z + b_3 A_{\varphi_x}\right| \quad (20c)$$

$$F_{3z} = F_{4z} = k_z\left|A_Z - b_2 A_{\varphi_x}\right| \quad (20d)$$

where A_Y, A_Z and A_{φ_x} have the expressions (2.4.12)-(2.4.14) from [5].

►*Excitation by pitching couple M_y*

●the elastic strains of the springs are:

$$u_{1x} = u_{2x} = \left(A_X - hA_{\varphi_y} - b_3 A_{\varphi_x}\right)\sin\omega t \quad (21a)$$

$$u_{3x} = u_{4x} = \left(A_X - hA_{\varphi_y} + b_2 A_{\varphi_x}\right)\sin\omega t \quad (21b)$$

$$u_{1y} = -u_{2y} = -u_{3y} = u_{4y} = aA_{\varphi_z}\sin\omega t \quad (21c)$$

$$-u_{1z} = u_{2z} = u_{3z} = -u_{4z} = aA_{\varphi_y}\sin\omega t \quad (21d)$$

●the modulus of maximum transmitted forces are

$$F_{1x} = F_{2x} = k_x\left|A_X - hA_{\varphi_y} - b_3 A_{\varphi_x}\right| \quad (22a)$$

$$F_{3x} = F_{4x} = k_x\left|A_X - hA_{\varphi_y} + b_2 A_{\varphi_x}\right| \quad (22b)$$

$$F_{1y} = F_{2y} = F_{3y} = F_{4y} = k_y a\left|A_{\varphi_z}\right| \quad (22c)$$

$$F_{1z} = F_{2z} = F_{3z} = F_{4z} = k_z a\left|A_{\varphi_y}\right| \quad (22d)$$

where A_X, A_{φ_y} and A_{φ_z} have the expressions (2.4.227)-(2.4.229) from [5].

►*Excitation by eccentric vertical force F_z*

●the elastic strains of the springs are:

$$u_{1x} = u_{2x} = \left(A_X - hA_{\varphi_y} - b_3 A_{\varphi_x}\right)\sin\omega t \quad (23a)$$

$$u_{3x} = u_{4x} = \left(A_X - hA_{\varphi_y} + b_2 A_{\varphi_x}\right)\sin\omega t \quad (23b)$$

$$u_{1y} = u_{4y} = \left(A_Y + aA_{\varphi_z} + hA_{\varphi_x}\right)\sin\omega t \quad (23c)$$

$$u_{2y} = u_{3y} = \left(A_Y - aA_{\varphi_z} + hA_{\varphi_x}\right)\sin\omega t \quad (23d)$$

$$u_{1z} = \left(A_Z + b_3 A_{\varphi_x} - aA_{\varphi_y}\right)\sin\omega t \quad (23e)$$

$$u_{2z} = \left(A_Z + b_3 A_{\varphi_x} + aA_{\varphi_y}\right)\sin\omega t \quad (23f)$$

$$u_{3z} = \left(A_Z - b_2 A_{\varphi_x} + aA_{\varphi_y}\right)\sin\omega t \quad (23g)$$

$$u_{4z} = \left(A_Z - b_2 A_{\varphi_x} - aA_{\varphi_y}\right)\sin\omega t \quad (23h)$$

●the modulus of maximum transmitted forces are

$$F_{1x} = F_{2x} = k_x\left|A_X - hA_{\varphi_y} - b_3 A_{\varphi_x}\right| \quad (24a)$$

$$F_{3x} = F_{4x} = k_x\left|A_X - hA_{\varphi_y} + b_2 A_{\varphi_x}\right| \quad (24b)$$

$$F_{1y} = F_{4y} = k_y \left| A_Y + aA_{\varphi_z} + hA_{\varphi_x} \right| \tag{24c}$$

$$F_{2y} = F_{3y} = k_y \left| A_Y - aA_{\varphi_z} + hA_{\varphi_x} \right| \tag{24d}$$

$$F_{1z} = k_z \left| A_Z + b_3 A_{\varphi_x} - aA_{\varphi_y} \right| \tag{24e}$$

$$F_{2z} = k_z \left| A_Z + b_3 A_{\varphi_x} + aA_{\varphi_y} \right| \tag{24f}$$

$$F_{3z} = k_z \left| A_Z - b_2 A_{\varphi_x} + aA_{\varphi_y} \right| \tag{24g}$$

$$F_{4z} = k_z \left| A_Z - b_2 A_{\varphi_x} - aA_{\varphi_y} \right| \tag{24h}$$

where A_X, A_Y, A_Z, A_{φ_x} A_{φ_y} and A_{φ_z} have the expressions (2.4.242)-(2.4.244), (2.4.252)-(2.4.254) from [5].

►*Excitation by eccentric longitudinal force F_y* – the analytical expressions of the elastic strains of the springs and of the maximum values of the transmitted forces are the same like those given by the relations (23a)-(23h) and (23a)-(23h), but the expressions of the amplitudes A_X, A_Y, A_Z, A_{φ_x} A_{φ_y} and A_{φ_z} may taken from the relations (2.4.265)-(2.4.267) and (2.4.275)-(2.4.277) from [5].

►*Excitation by inclined longitudinal force F* – the analytical expressions of the elastic strains and of the maximum values of the transmitted forces are those given by the relations (23a)-(23h) and (23a)-(23h), where the amplitudes A_X, A_Y, A_Z, A_{φ_x} A_{φ_y} and A_{φ_z} are given in [5] by the relations (2.4.306)-(2.4.308) and (2.4.316)-(2.4.318).

4. CONCLUSIONS

▪thanks to the linearity of the mechanical system and to the nature of the excitations (harmonical), the steady-state forced vibration of the rigid body are harmonic too, that meaning harmonical vibration for all points of it;

▪considering the behavior of the bearings linear elastic, the forced vibration of the rigid body are in phase or in opposite phase with the harmonical excitations, the same time variation having the elastic strains;

▪this study is determining the expressions of the three directions components of the transmitted forces to the foundation trough the four elastic bearings; in order to establish the modulus of the vector forces transmitted to the foundation, it have to use the relation (3).

5. BIBLIOGRAPHIE

[1]Bratu, P., Drăgan, N. – *“L'analyse dynamique de l'interaction machine-structure sur la base du modèle equivalent de rigide aux liaisons visco-elastiques”*, Analele Universității “Dunărea de Jos” Fascicula XIV, Galați, 1997

[2]Bratu, P., Drăgan, N. – *“L'analyse des mouvements désaccouplés appliquée au modèle de solide rigide aux liaisons élastiques”*, Analele Universității “Dunărea de Jos” Fascicula XIV, Galați, 1997

[3]Bratu, P. – *“Vibrațiile sistemelor elastice”*, Editura Tehnică, Bucureşti, 2000

[4]Bratu, P. – *”Sisteme elastice de rezemare pentru maşini şi utilaje”*, Editura Tehnică, Bucureşti, 1990

[5]Drăgan, N. – *“Contribuții la analiza şi optimizarea procesului de transport prin vibrații”* teză de doctorat, Universitatea “Dunărea de Jos”, Galați, 2001